

IMP	Indicators	Students might think or do:
<p><b>1: Make sense of problems and persevere in solving them</b></p>	<ul style="list-style-type: none"> <li>✓ explain to themselves the meaning of a problem</li> <li>✓ look for entry points to its solution</li> <li>✓ analyze givens, constraints, relationships, and goals</li> <li>✓ make conjectures about the form and meaning of the solution</li> <li>✓ plan a solution pathway</li> <li>✓ consider analogous problems</li> <li>✓ monitor and evaluate progress and change course if necessary.</li> </ul>	<ul style="list-style-type: none"> <li>▪ "I tried that approach to solving the problem and it didn't work. What's another way I can try to solve it?"</li> <li>▪ "What's a useful way to begin working on this problem?"</li> <li>▪ They can set up a series of steps to follow to get themselves to the answer.</li> <li>▪ "There's another problem I've done that's like this that might help me here."</li> <li>▪ "This isn't working; I need to try something else."</li> </ul>
<p><b>2: Reason abstractly and quantitatively</b></p>	<ul style="list-style-type: none"> <li>✓ make sense of quantities and their relationships in problem situations</li> <li>✓ <i>decontextualize</i> -- abstract and represent a problem situation symbolically and manipulate those symbols without attending to their referents</li> <li>✓ <i>contextualize</i> -- pause during problem solving to connect symbolic work back to the context of the problem</li> <li>✓ Pay attention to the important quantities and relationships between them</li> <li>✓ use representations to highlight those relationships and the underlying mathematical structure of a problem</li> </ul>	<ul style="list-style-type: none"> <li>▪ "How can I capture important information in a diagram or model?"</li> <li>▪ "What solution path does this diagram or model imply?"</li> <li>▪ "OK, I've done all these calculations; now, what does that mean in the problem? Does my answer make sense for answering this problem?"</li> <li>▪ Given the problem: <i>There are <math>\frac{3}{5}</math> as many boys as girls. If there are 45 boys, how many girls are there?</i> a student can create a diagram that shows the relationship between the number of boys, of girls, and of all the children together.</li> </ul>
<p><b>3: Construct viable arguments and critique the reasoning of others</b></p>	<ul style="list-style-type: none"> <li>✓ make conjectures and build a logical progression of statements to explore the truth of their conjectures</li> <li>✓ analyze situations by breaking them into cases</li> <li>✓ recognize and use counterexamples</li> <li>✓ justify conclusions, communicate them to others, and respond to the arguments of others</li> <li>✓ distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is</li> </ul>	<ul style="list-style-type: none"> <li>▪ A student can state a rule for a pattern, and can explain why their rule works for that pattern.</li> <li>▪ When someone claims "multiplying two numbers gives you an answer bigger than either the numbers," a student can think about:             <ul style="list-style-type: none"> <li>-- what happens when you multiply 2 whole numbers;</li> <li>-- what happens when you multiply by a fraction;</li> <li>-- what happens when you multiply 2 fractions as separate possibilities to consider.</li> </ul> </li> </ul>

<p><b>4: Model with mathematics</b></p>	<ul style="list-style-type: none"> <li>✓ make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later</li> <li>✓ identify important quantities in a practical situation</li> <li>✓ map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas</li> <li>✓ analyze those relationships mathematically to draw conclusions</li> <li>✓ interpret their mathematical results in the context of the situation</li> </ul>	<ul style="list-style-type: none"> <li>▪ When starting to solve a problem, a student makes a table, a flowchart or graphs the data to see if that representation will shed light on the problem solution.</li> <li>▪ When working on a problem, a student might estimate or round off a certain quantity <i>for the purpose of moving through the calculation to see what the results are</i> – knowing they will have to readjust to be more precise.</li> </ul>
<p><b>5: Use appropriate tools strategically</b> (Ex: pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software.)</p>	<ul style="list-style-type: none"> <li>✓ are familiar with tools appropriate for their grade or course and can make sound decisions about when each of these tools might be helpful</li> <li>✓ identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems</li> </ul>	<ul style="list-style-type: none"> <li>▪ A student wants to see how the difference between values in a table changes, so she begins by making a table, then decides to put the information in a spreadsheet to more easily do the calculations, and draw conclusions from the results.</li> <li>▪ A student is having trouble visualizing a situation with a geometric shape, so he creates it in a geometry software application and is able to move the shape around to see how some parts of the shape change while keeping certain characteristics of the shape the same.</li> </ul>
<p><b>6: Attend to precision</b></p>	<ul style="list-style-type: none"> <li>✓ use clear definitions in discussion with others and in their own reasoning</li> <li>✓ state the meaning of the symbols they choose, including using the equal sign consistently and appropriately</li> <li>✓ are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem</li> <li>✓ express numerical answers with a degree of precision appropriate for the problem context</li> </ul>	<ul style="list-style-type: none"> <li>▪ A student rewrites his explanation to a problem using appropriate mathematics vocabulary.</li> <li>▪ A student learns why it is incorrect to write <math>14 + 4 = 18 + 5 = 23 \times 2 = 46</math>.</li> <li>▪ "My calculator says 3.581279, but since I'm asked to find the number of inches, that's not a number that makes sense to write for measurement in inches. I'll say 3.5 or 3.6."</li> </ul>

<p><b>7: Look for and make use of structure</b></p>	<ul style="list-style-type: none"> <li>✓ look for similar mathematical structures across seemingly different problems</li> <li>✓ use those similarities to help them reason about how to solve a problem</li> <li>✓ can step back for an overview and shift perspective</li> <li>✓ can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects</li> </ul>	<ul style="list-style-type: none"> <li>▪ “Since (problem A) and (problem B) are structurally the same, what do I know about solving (prob A) that will help me think about solving (prob B)?”</li> <li>▪ “Figuring out what to do with <math>3(x + 2)</math> is just like the work I did in (?th) grade when I learned that <math>7 \times 8</math> is the same as <math>(7 \times 3) + (7 \times 5)</math>.”</li> <li>▪ Would recognize that <math>17 + 2(X + 1)</math> will be odd because <math>2(x+1)</math> is even (since it’s 2 times some number) and 17 is odd, and an odd amount + an even amount will be odd.</li> <li>▪ Noticing that all numbers that have a remainder of 4 when divided by 5 will end in either 4 or 9</li> <li>▪ “I recognize that <math>\frac{1}{3}(A + B + C)</math> is really just the same as finding the average of 3 numbers.”</li> </ul>
<p><b>8: Look for and express regularity in repeated reasoning.</b></p>	<ul style="list-style-type: none"> <li>✓ look both for general methods and for shortcuts in the calculations, understanding why the shortcuts work</li> <li>✓ maintain oversight of the problem--solving process, while attending to the details of the calculations</li> <li>✓ continually evaluate the reasonableness of their intermediate results</li> </ul>	<ul style="list-style-type: none"> <li>▪ “When I divide 15 by 9, the 9 keeps ‘going in’ 6 times .... over and over again. That means I have a repeating decimal.”</li> <li>▪ “I solve this problem using 8 adults. Then I solved it using 10 adults, 12 adults and 20 adults. What’s the same about my solution steps each time? How can that help me describe a process or an equation for the problem?”</li> <li>▪ (When a student is immersed in some calculations, they can stop and think) “Wait, where am I going with this? What does 5.76 represent? Where am I in the process of solving this problem?”</li> <li>▪ “Wait, I can’t just write 7.2 because you can’t have 7.2 children in a group.”</li> </ul>